

# Group Theory in Subnuclear Physics

---

**FI. STANCU**

*Institute of Physics, University of LiPge*

**CLARENDON PRESS • OXFORD**

# CONTENTS

<b>1</b>	<b>Symmetries in quantum mechanics</b>	<b>1</b>
1.1	The role of group theory	1
1.2	Types of symmetries	2
1.3	Invariance and conservation laws	6
<b>2</b>	<b>Elements of group theory</b>	<b>9</b>
2.1	Definition of a group (of transformations)	9
2.2	Subgroups	16
2.3	Isomorphism and homomorphism	18
2.4	Cayley's theorem	20
2.5	Classes	22
2.6	Simple and semi-simple groups	25
2.7	Direct product groups and semi-direct product groups	27
<b>3</b>	<b>Linear representations of a group</b>	<b>29</b>
3.1	Linear vector spaces	29
3.2	Definition of a group representation	31
3.3	Matrix representations	32
3.4	Equivalent representations	32
3.5	Unitary representations	33
3.6	Characters	34
3.7	Examples of representations	35
3.8	Irreducible representations	39
3.9	Direct products of representations	42
3.10	Schur's lemmas and the orthogonality theorem	43
3.11	The regular representation	48
3.12	Construction of character tables	53
3.13	Clebsch-Gordan series	57
<b>4</b>	<b>Permutation group</b>	<b>59</b>
4.1	General remarks	59
4.2	Irreducible representations	60
4.3	Basis functions of $S_n$	62
4.4	Matrices of irreducible representations	89
4.5	The tensor method	103
4.6	Outer products	108
4.7	Inner products. Clebsch-Gordan series and coefficients	112
4.8	More about Clebsch-Gordan coefficients of $S_n$	119
*4.9	The K-matrix (isoscalar factor)	128

*4.10	The TC-matrix	131
4.11	Baryons as three-quark states	140
*4.12	Braid groups and new developments	144
<b>5</b>	<b>Lie groups</b>	<b>148</b>
5.1	Infinitesimal transformations	149
5.2	Structure constants	151
5.3	Generators	154
5.4	Simple and semi-simple groups	158
5.5	Simple and semi-simple Lie algebras	162
5.6	Examples of Lie groups	163
5.7	Compactness	169
*5.8	Direct and semi-direct sums of Lie algebras	170
5.9	Classification of semi-simple groups	171
5.10	Representations of semi-simple groups	192
5.11	The tensor method (revisited)	198
*5.12	Quantum groups	204
<b>6</b>	<b>The orthogonal group</b>	<b>209</b>
6.1	The rotation group $R_3$ or $SO(3)$	210
6.2	The group $O(4)$	230
6.3	The Euclidean groups	239
<b>7</b>	<b>The Poincaré group and the Lorentz group</b>	<b>241</b>
7.1	Notation	241
7.2	Lorentz transformations	242
7.3	Infinitesimal transformations	248
7.4	The spin of a Dirac particle	250
7.5	Irreducible representations	254
7.6	The Poincaré group	256
<b>8</b>	<b>Unitary groups</b>	<b>262</b>
8.1	General properties	263
8.2	The group $SU(2)$	265
8.3	The homomorphism of $SU(2)$ with $R_3$	266
8.4	Multiplets	269
8.5	G-parity	271
8.6	The group $SU(3)$	274
8.7	Beyond $SU(3)$	315
8.8	Heavy flavours	321
8.9	The adjoint representation	337
8.10	The tensor method	340
8.11	Clebsch-Gordan coefficients for $SU(3)$	344

<b>*9 Gauge groups</b>	<b>347</b>
9.1 Abelian gauge symmetry	347
9.2 Non-abelian gauge symmetry	349
<b>*10 Multiquark systems</b>	<b>355</b>
10.1 The dynamics	356
10.2 The baryons	360
10.3 Diquonia	372
10.4 Six-quark systems	383
Appendix A. Conservation laws	398
Appendix B. The rearrangement theorem, Schur's lemmas, and the orthogonality theorem	402
Appendix C. Invariant integration	407
Appendix D. Dimension of an $SU(n)$ irrep	410
References	413
Index	419